

## Layer thickness estimation from the frequency spectrum of seismic reflection data

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### Summary

We compare the spectra of Short Time Window Fourier Transform (STFT) and Constrained Least Squares Spectral Analysis (CLSSA) for spectral minima periodicity. Using time thickness equals  $1/df$ , where  $df$  is frequency period, we show that spectral minima approach using CLSSA gives more accurate time thicknesses than STFT for the same analysis window.

Starting with a broad band synthetic seismic data generated from a wedge model, we extract selected traces at known temporal thicknesses for time frequency analysis. We cross plot apparent time thicknesses derived from CLSSA and STFT line spectra using the approach above, against true time thicknesses of the wedge model. The result shows apparent CLSSA thicknesses that are strongly correlated with true time thicknesses

We extend this study to real seismic data from Hitts Lake Field, onshore Texas and show that the results are consistent with the results from model studies.

### Introduction

For a given pair of reflection coefficients separated by a time thickness,  $T$ , Marfurt and Kirlin (2001) show the frequency impulse response to consist of periodic spectral maxima and minima. The time separation  $T$ , between the reflection pair can be determined by  $1/df$ , where  $df$  is the frequency period or frequency separation between two spectral minima. This technique is the basis of layer thickness estimation using spectral decomposition (e.g. Partyka et al., 1999).

The accuracy of layer thickness estimation using this rule, i.e. time thickness equals  $1/df$ , may depend on the resolution of the spectral decomposition method utilized in transforming the seismic data from time to frequency domain. This is the case with the Short Time Window Fourier Transform (STFT) and the Constrained Least Squares Spectral Analysis (CLSSA).

The objective of this paper is to compare the accuracy of layer thickness estimation using these spectral decomposition techniques

### Theory

CLSSA (Puryear et al., 2012) is an inversion based spectral decomposition that computes the spectrum of a windowed segment of a seismic trace by finding a sparse weighted summation of truncated sinusoids that reconstruct the signal. The sinusoids weights correspond to the Fourier series coefficients. The method properly gives the spectrum of the data within the window rather than the spectrum of the windowed data. The forward problem is given as:

$$G = md \quad (1)$$

where  $G$  is the kernel matrix,  $m$  is the desired spectral decomposition coefficients and  $d$  is the input seismic trace.

STFT also known as the sliding window Discrete Fourier Transform (e.g. Cohen, 1994) computes the Fourier spectrum of a trace by sliding a time window along the trace. The resulting spectrum is the Fourier Transform of the data within the window convolved with the spectrum of the window (equation (2)). This approach yields time frequency analysis that has the time resolution of the window. Ideally, shorter lengths of decomposition windows are desirable in order to increase temporal resolution.

$$STFT(\tau, \omega) = \int f(t)g(t - \tau)e^{-j\omega t} dt \quad (2)$$

Where  $f(t)$  is the input seismic trace and  $g(t)$  is a function of the decomposition window.

However, as the analysis window progressively becomes shorter STFT tends to smear spectral energy across the frequency band, a phenomenon often referred to as spectral leakage (e.g. Bracewell, 1986). This phenomenon follows from the Fourier Similarity theorem sometimes referred to as the Heisenberg Uncertainty Principle that explains improved time resolution to be the expense of poorer frequency resolution and vice versa. For example, Figure 3 is a time frequency analysis of a broad band synthetic seismic data generated from a wedge model (Figure 1). Note the high spectral energy on the 10 ms analysis window of STFT spectrum (indicated by the block white arrows). The Fourier amplitude spectrum of the same trace (Figure 2) does not show this high energy at the indicated frequencies but rather, shows negligible spectral energy at the designated frequency band. This energy is fictitious and originates from other frequencies, smeared across and beyond the seismic bandwidth. This effect is less pronounced for longer windows (e.g. 40 ms and 80 ms windows of Figure 3) Conversely, CLSSA seems to maintain fairly uniform Gaussian distributed spectrum regardless of the length of the decomposition window.

## Layer thickness estimation

### Method

We set out to show the effect of smearing of STFT spectrum on layer thickness estimation by modeling a sand wedge encased in shale. This model setup, achieves equal and opposite reflection coefficients. We generate a broad band synthetic seismic data (Figure 1) from this model using a 100 Hz Ricker wavelet. The choice of a broad band synthetic data is to allow adequate frequencies to resolve periodic spectral minima or ‘spectral notches’ in the amplitude spectrum. We analyze several traces from common depth point (CDP) locations at variable time thicknesses of the wedge model (Table 1).

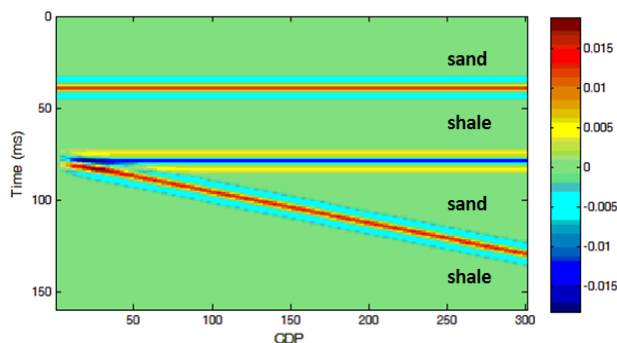


Figure 1: Synthetic broad band seismic data for a wedge sand model encased in shale.

CDP	Time thickness (ms)	Wavelet fraction ( $v=f\lambda$ )	Analysis window (ms)
20	5.0	$\sim \lambda/4$	10
27	6.8	$\sim \lambda/3$	15
35	8.9	$\sim \lambda/2$	20
78	19.9	$\sim \lambda$	40
157	39.9	$\sim 2\lambda$	80

Table 1: CDP locations of synthetic seismic traces analyzed for STFT and CLSSA using several analysis windows.

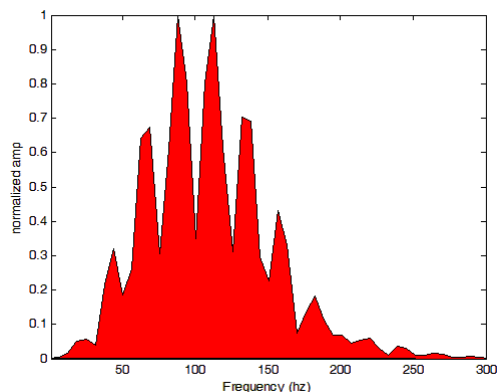


Figure 2: Fourier amplitude spectrum of seismic trace extracted from CDP 20

At these CDP locations, time thicknesses correspond to wavelength fractions of the Ricker wavelet, computed from the velocity of the sand wedge and the dominant wavelet frequency (e.g Widess, 1973). Our goal is to test the accuracy of STFT and CLSSA spectra in determining known time thicknesses at these locations. Figure 3 is the time frequency panel of selected traces in Table 1, analyzed for STFT and CLSSA using analysis windows that are about twice the time thicknesses of the wedge at the trace locations. The temporal thicknesses of the wedge model at these trace locations are given in Table 1. Using equation (3), we compute the frequency impulse response (e.g. Marfurt and Kirilin, 2001) that will be generated by a layer of equivalent time thickness and reflectivity as that of the wedge model at the given CDP location. For comparison, we multiply this impulse response by the spectrum of the wavelet (wavelet spectrum in Figure 4) to produce an analytical spectrum.

$$G(f) = r_1(\theta) \exp(-i2\pi f t_1) + r_2(\theta) \exp(-i2\pi f [t_1 + T]) \quad (3)$$

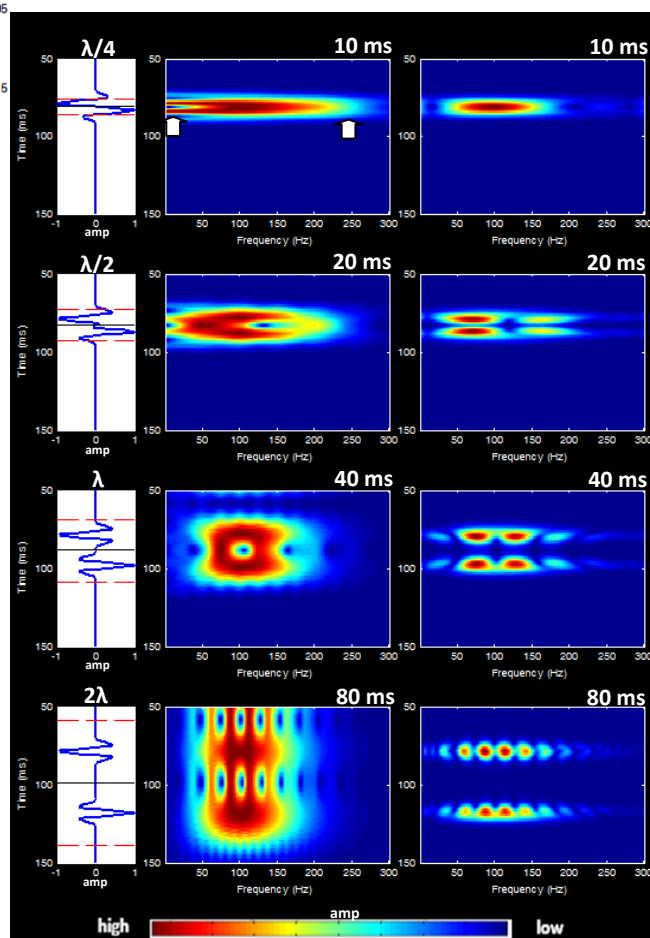


Figure 3: Time frequency analysis of selected traces in Table 1. The dashed red lines on the trace panel indicate position of analysis window with respect to the center of the trace. The solid black line indicates the time-line along which line spectrum is drawn.

### Layer thickness estimation

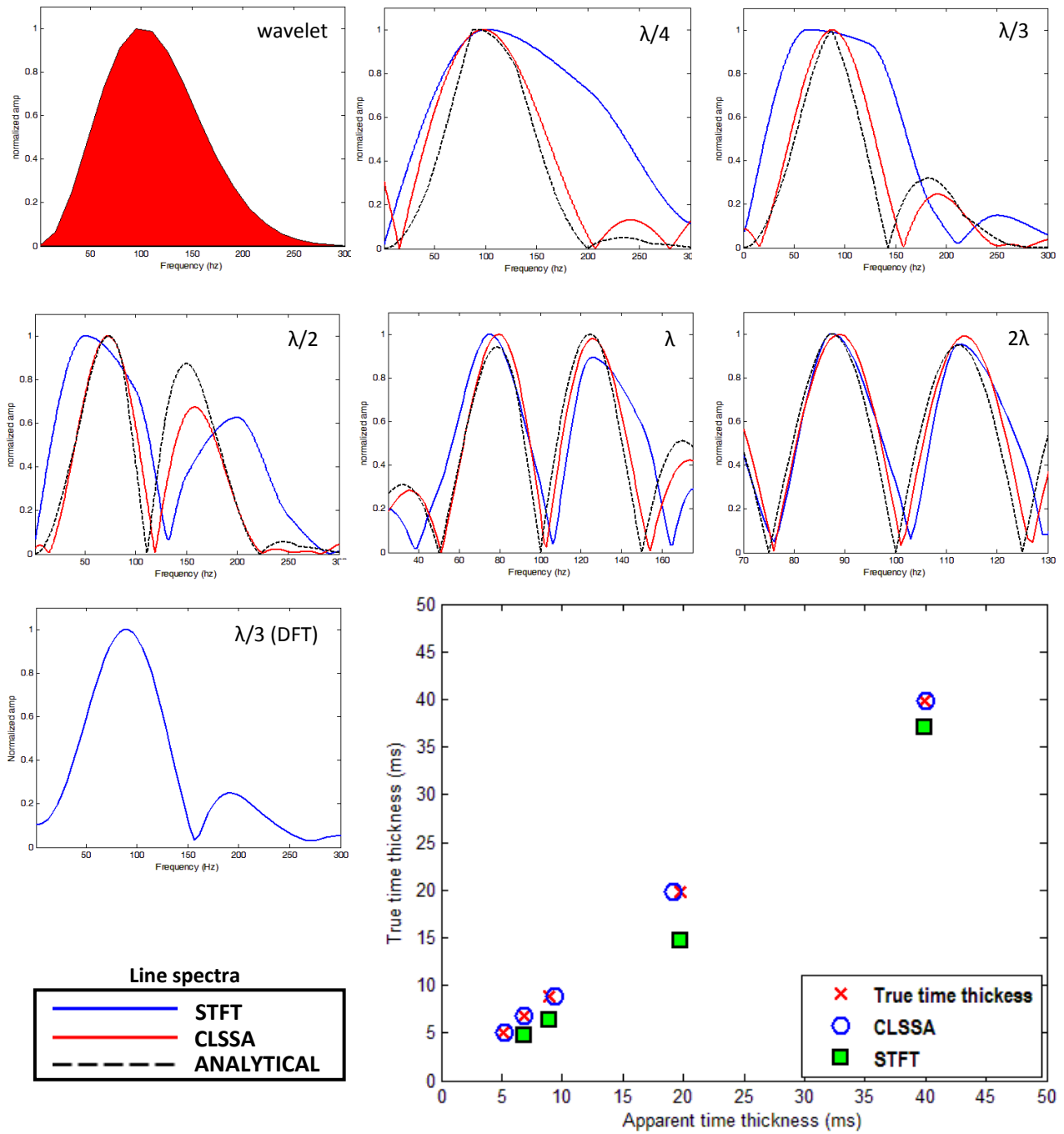


Figure 4: Wavelet spectrum, line spectra and cross-plot of apparent time thickness derived from CLSSA and STFT versus true time thickness

## Layer thickness estimation

We compare this analytical spectrum with the line spectra of STFT and CLSSA drawn along the center of the trace (indicated by solid black line on the trace panel of Figure 3). Results of the comparison are shown in Figure 4. We proceed to determine apparent time thickness from the line spectra by applying time thickness equals  $1/df$ , where  $df$  is the frequency separation between two spectral notches. The spectral notch periodicity from CLSSA line spectrum seems to match the analytical spectrum well for all time thicknesses analyzed. This is evident on the cross plot of apparent time thickness derived from CLSSA spectrum versus true time thickness of wedge model (Figure 4). STFT on the other hand does not seem to match the analytical spectrum for shorter analysis windows (e.g. for  $\lambda/4$ ,  $\lambda/3$  and  $\lambda/2$ ) because of the effect of spectral smearing, unlike CLSSA that properly gives the spectrum of the data within the window rather than the spectrum of the windowed data. For longer analysis window, where the effect of spectral smearing is less pronounced, STFT spectral notch periodicity begins to fit the analytical spectrum. Using large analysis windows for time frequency analysis can increase the chances of non isolated reflection events to be captured within the window. This may degrade the spectrum.

The cross plot result for STFT line spectra shows underestimation of true time thicknesses. To test this observation we compute the Discrete Fourier Transform of an equivalent trace length as that within the analysis window of  $\lambda/3$ . The result (labeled  $\lambda/3(\text{DFT})$  of Figure 4) shows notch periodicity that seems to agree with the analytical spectrum and CLSSA spectrum of  $\lambda/3$  (of Figure 4). We attribute this time thickness underestimation from spectral notch periodicity using STFT amplitude spectrum to be the consequence of spectral smearing

We extend this study to real seismic data from Hitts Lake Field, onshore Texas. This field is a case of sand reservoirs with shale intercalations, which is similar to the wedge model scenario above. Figure 5(i) is a correlation panel of seismic and well data. A correlation coefficient of 0.9 is achieved for this tie. The red trace is a synthetic trace generated from well data and the blue is a real composite trace extracted from seismic data at the well location.

Our interest in this trace is the reflection event indicated by the green arrow, which approximates a reflection of equal and opposite coefficients. Figure 5(ii) is a time frequency panel for the real trace segment analyzed with a 30 ms window. Figure 5(iii) is line spectra drawn along the center of the trace segment (indicated by dashed white line). CLSSA gives notch frequency period of 46 Hz, which translates to 21.73 ms. Using average interval velocity of 3048 m/s, this apparent time thickness gives a thickness of 108.69 ft ( $v=2\lambda/t$ ). From well data the thickness of this bed is interpreted to be 109 ft. STFT because of spectral smearing yields notch frequency period of about 94 Hz, corresponding to a thickness estimate of 54 ft. This is about half the true thickness (109 ft) from well data.

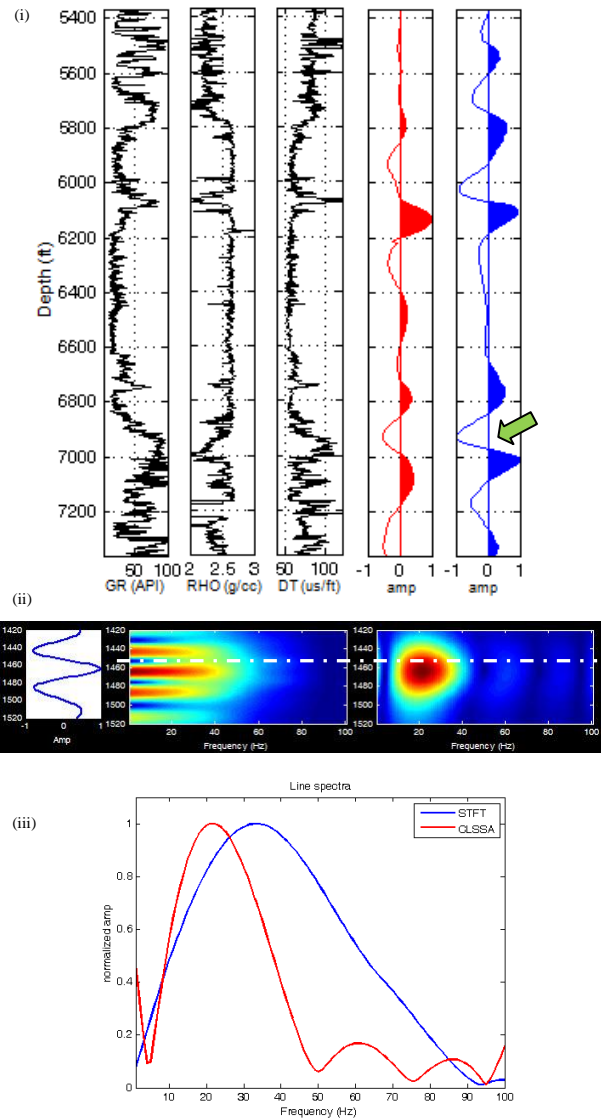


Figure 5: (i) well to seismic tie. (ii) Time frequency panel. (iii) Line spectra

## Conclusions

We have shown from synthetic seismic data and real seismic data using spectral notch periodicity approach, the tendency of layer thickness underestimation using STFT spectrum, as a result of spectral smearing. Conversely, for the same analysis window, CLSSA using this approach yields more accurate layer thickness estimation.

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#### **EDITED REFERENCES**

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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